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TRIGONOMETRIK FUNKSIYALARNI EKVIVALENT TA'RIFI HAQIDA

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ANNOTATSIYA

Ushbu maqolada trigonometrik funksiyalarni geometric ta'rifidan farqli bo'lgan qatorlar orqali ta'rifi o'rganilgan. Turli matematik tushunchalarning ekvivalent ta'rifi bo'lgani sababli, trigonometrik funksiyalarning boshqa ta'riflari ham mavjud. Geometrik ta'rifidagi xossalari qatorlar orqali ta'rifining xossalari ustma-ust tushishi ko'rsatilgan.Xuddi shu kabi tgx;ctgx;secx va cosecx funksiyalar uchun ham qatorlar orqali ta'rifni berish mumkin.

Kalit so'zlar. *Trigonometrik funksiyalar, Teylor qatori, formula, xosila, integral, funksiyaning analitik ko'rinishlari,darajali qatorlar.*

АННОТАЦИЯ

В трех статьях изучается определение тригонометрических функций рядами, отличное от геометрического определения. Поскольку существуют эквивалентные определения различных математических операций, существуют и другие определения тригонометрических функций. Свойства геометрического определения уникальны тем, что они перекрываются со свойствами определения через линии.Аналогично определение функций tgx;ctgx;secx и cosecx может быть задано через линии.

Ключевые слова. *Тригонометрические функции, ряд Тейлора, формула, произведение, целые функции, аналитические представления, степенные ряды.*

ABSTRACT

In this article, the definition of trigonometric functions through lines, which is different from the geometric definition, is studied. Since different mathematical concepts have equivalent definitions, there are other definitions of trigonometric functions. It is shown that the properties of the geometric definition are superimposed on the properties of the definition through lines. In the same way, the definition of the functions tgx;ctgx;secx and cosecx can be given through lines.

Key words. *Trigonometric functions, Taylor's series, formula, product, integral, analytic representations of the function, power series.*

KIRISH

Matematik tushunchalarni o'zlashtirish ko'p faktorlarga bog'liq. Ana shu faktorlardan biri tushunchalarni har xil ta'riflarini o'zlashtirish va muayyan ta'rifga asoslangan holda unga mos nazariyani o'zlashtirishdir. Muayyan tushuncha uchun mavjud bo'lgan har xil talqinlar uning mohiyatini, mazmunini va hajmini o'zlashtirish imkonini beradi.

Ma'lumki, trigonometrik funksiyalar geometrik tarzda ta'rif berilgan. Biroq fanda geometriyaga bog'liq bo'lmagan holda trigonometrik funksiyalarni o'rganish zaruriyati vujudga keldi. Matematik analiz kursidan ma'lumki bu funksiyalarni darajali qatorga yoyish mumkinligi haqida gapiriladi, ya'ni:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

Ammo mazkur tengliklarning o'ng tarafidagi qatorlarni asos qilib olgan holda trigonometrik funksiya tushunchalarini kiritish va shu asosda uning nazariyasini rivojlantirish mumkin.

Ta'rif. Analitik $\cos x$ va $\sin x$ funksiyalar deb, quyidagi formulalar bilan berilgan funksiyaga aytiladi:

$$c(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$s(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Mana shu ta'rifdan kelib chiquvchi $c(x)$ va $s(x)$ funksiyalarni asosiy xossalarini keltiramiz.

1. $c(x)$ va $s(x)$ funksiyalarni aniqlanish sohasi barcha haqiqiy sonlar to'plamidir. Huddi shuningdek $c(x)$ va $s(x)$ darajali qator, x ning haqiqiy o'zgaruvchili qiymatida yaqunlashadi. Bunga ishonch hosil qilish uchun Dalamberning absolyut yaqinlashishi prinsipini qo'llash yetarlidir. Shuning uchun $c(x)$ qator uchun

$$\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right| = \lim_{n\to\infty}\left|\frac{x^{2(n+1)}(2n)!}{[2(n+1)]!\,x^{2n}}\right| = \lim_{n\to\infty}\frac{|x|^2}{(2n+1)(2n+2)} = 0 < 1.$$

2. $c(x)$ funksiya juft , $s(x)$ esa toq , ya'ni

$c(-x) = c(x), \;\; s(-x) = -s(x)$

3. $c(x)$ va $s(x)$ funksiyalar uchun qo'shish formulalari:

$c(x - y) = c(x)\, c(y) + s(x)\, s(y)$

$s(x + y) = s(x)\, c(y) + c(x) s(y)$

Bu formulalarni isbotlash uchun qatorlar ustida amallarni bajarib ko'rsatish mumkin.

Natija:

1) $y = x$ bo'lsa, $c^2(x) + s^2(x) = 1$ ni hosil qilish mumkin.

2) y ni $-y$ ga almashtirib ,

$c(x+y) = c(x)\,c(y) - s(x)\,s(y)$

$s(x-y) = s(x)\,c(y) - c(x)\,s(y)$

ni hosil qilamiz.

4. Ko'paytmani yig'indiga almashtirish formulalari:

$$c(x)\,c(y) = \frac{c(x+y) - c(x-y)}{2}$$

$$s(x)\,s(y) = \frac{c(x-y) - c(x+y)}{2}$$

$$c(x)\,s(y) = \frac{s(x+y) - s(x+y)}{2}$$

5. Yig'indini ko'paytmaga almashtirish formulalari:

$$c(x) + c(y) = 2c\left(\frac{x+y}{2}\right)\,c\left(\frac{x-y}{2}\right)$$

$$c(x) - c(y) = -2s\left(\frac{x+y}{2}\right)\,s\left(\frac{x-y}{2}\right))$$

$$s(x) + s(y) = 2s\left(\frac{x+y}{2}\right)\,c\left(\frac{x-y}{2}\right)$$

$$s(x) - s(y) = 2c\left(\frac{x+y}{2}\right)\,s\left(\frac{x-y}{2}\right)$$

6. Ikkilangan argument va yarim argument formulalari:

$$s(2x) = 2s(x)\,c(x), \quad c(2x) = c^2(x) - s^2(x),$$

$$c\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + c(x)}{2}}, \qquad s\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - c(x)}{2}}$$

7. $c(x)$ va $s(x)$ funksiyalarning hosilalari:

$$c'(x) = -x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = -s(x)$$

$$s'(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = c(x)$$

8. $s(x-y) = s(x)\,c(y) - c(x)\,s(y)$

$c(x-y) = c(x)\,c(y) + s(x)\,s(y)$

formulada 1) $x = 0\,,\ y = \frac{\pi}{2}$ 2) $x = y$ 3) $x = \frac{\pi}{2},\ y = 0$ qiymatlarda.

$$s\left(-\frac{\pi}{2}\right) = -s\left(\frac{\pi}{2}\right) = s(0)\,s\left(\frac{\pi}{2}\right) - c(0)\,s\left(\frac{\pi}{2}\right)$$

$$c\left(-\frac{\pi}{2}\right) = c(0)\, c\left(\frac{\pi}{2}\right) + s(0)\, s\left(\frac{\pi}{2}\right)$$
$$s(0) = s^2(x)\, c^2(y) - c(x)\ \ s(y) = 0$$
$$c(0) = c^2(x) + s^2(y)$$
$$s\left(\frac{\pi}{2}\right) = s\left(\frac{\pi}{2}\right)\ c(0) - c\left(\frac{\pi}{2}\right)\ \ s(0)$$
$$c\left(\frac{\pi}{2}\right) = c\left(\frac{\pi}{2}\right)\ c(0) + s\left(\frac{\pi}{2}\right)\ \ s(0)$$

bo'lib, $s(0) = 0$, $c(0) = 1$, $s\left(\frac{\pi}{2}\right) = 1$, $c\left(\frac{\pi}{2}\right) = 0$ larni hosil qilish mumkin.

$$s(x+y) = s(x)\, c(y) + c(x) s(y)$$
$$c(x+y) = c(x)\, c(y) + s(x)\, s(y)$$

formulaga $y = \left(\frac{\pi}{2}\right)$ deb olsak.

$s\left(x + \frac{\pi}{2}\right) = c(x)$, $c\left(x + \frac{\pi}{2}\right) = -s(x)$ ni hosil qilamiz.

Bundan , $s(x+\pi) = c\ \left(x + \frac{\pi}{2}\right) = -s(x)$,

$c(x+\pi) = -s\left(x + \frac{\pi}{2}\right) = -c(x)$ kelib chiqadi.

9. $s(x)$ va $c(x)$ funksyalarning davriyligi

$$s(2\pi + x) = s[\pi + (\pi + x)] = -s(\pi + x) = s(x)$$
$$c(2\pi + x) = c[\pi + (\pi + x)] = -c(\pi + x) = c(x)$$

10. $s(x)$ va $c(x)$ funksyalar uzluksizdir.

$$\lim_{\Delta x \to 0} \frac{s(x+\Delta x) - s(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2s\left(\frac{\Delta x}{2}\right) c\left(x + \frac{\Delta x}{2}\right)}{\Delta x} = 0$$
$$\lim_{\Delta x \to 0} \frac{c(x+\Delta x) - c(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-2s\left(\frac{\Delta x}{2}\right) s\left(x + \frac{\Delta x}{2}\right)}{\Delta x} = 0$$

XULOSA

Shuningdek, boshqa trigonometrik funksiyalarni xossalarini o'rganish mumkin va $sinx$, $cosx$ funksiyalar aynan $s(x)$, $c(x)$ funksiyalar bilan ustma-ust tushadi, ya'ni $sinx \equiv s(x)$, $cosx \equiv c(x)$.

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