O'ZARO TESKARI FUNKSIYALARDAN OLINGAN INTEGRALLAR ORASIDAGI BOG'LANISH

Djabbarov Odil Djurayevich
TDTU Olmaliq filiali katta o'qituvchisi
odilxon455@gmail.com
Xujayev Tuymurod Xuddiyevich
TDTU Olmaliq filiali katta o'qituvchisi
tuymurod. khujayev@gmail.com

ANNOTATSIYA

Ushbu maqolada funksiyadan olingan integralni, bu funksiyaga teskari bo'lgan funksiyadan olingan integral orasidagi munosabat o'rganilgan. Bo'laklab integrallash formulasining umumlashmasi keltirilgan.

Kalit so'zlar: *Integral, belgilash usuli, bo'laklab integrallash usuli, xosila, yuqori tartibli xosila, boshlang'ich funksiya, figuraning yuzi, teskari funksiya.*

АННОТАЦИЯ

В данной статье исследуется связь между интегралом, полученным от функции, и интегралом, полученным от функции, обратной к этой функции. Дано обобщение формулы интегрирования по кусочкам.

Ключевые слова: *Интеграл, метод определения, метод кусочного интегрирования, производная, производная высшего порядка, начальная функция, грань фигуры, обратная функция.*

ABSTRACT

In this article, the relationship between the integral obtained from the function and the integral obtained from the function that is the inverse of this function is studied. A generalization of the formula for integration by pieces is given.

Key words: *Integral, definition method, piecewise integration method, derivative, higher-order derivative, initial function, face of a figure, inverse function.*

KIRISH

Bizga qandaydir $\int f(x)\,dx$ integral berilgan bo'lsin. Bu integralni hisoblash uchun turli usullar mavjud bo'lib, ulardan asosan ikki usul: belgilash va bo'laklab integrallash usullari muhim ahamiyatga ega. Shulardan belgilash usuliga to'g'ri keladigan ayrim integrallarni ko'rib chiqaylik:

1) $\int \frac{f'(x)}{f(x)}dx = \int \frac{d[f(x)]}{f(x)} = \ln[f(x)] + C$

2) $\int f'(x)\, f(x)dx = \int f(x)\, d[f(x)] = \frac{f^2(x)}{2} + C$

3) $\int f[\varphi(x)]\, \varphi'(x)dx = \int f[\varphi(x)]\, d[\varphi(x)] = F[\varphi(x)] + C$

Bu integrallar yordamida ko'plab misollarni tuzish mumkin. Masalan,

1) $\int \frac{\cos x}{\sin x} dx$ 2) $\int \frac{dx}{\sqrt{1-x^2}\arcsin x}$

Bo'laklab integrallashga doir ayrim misollarni ko'rib chiqaylik.

1) $\int xf'(x)dx = \int x\, d\,[f(x)] = \begin{vmatrix} x = u, & d[f(x)] = dv \\ dx = du, & v = f(x) \end{vmatrix} = x\,f(x) - \int f(x)dx = x\,f(x) - F(x) + C$

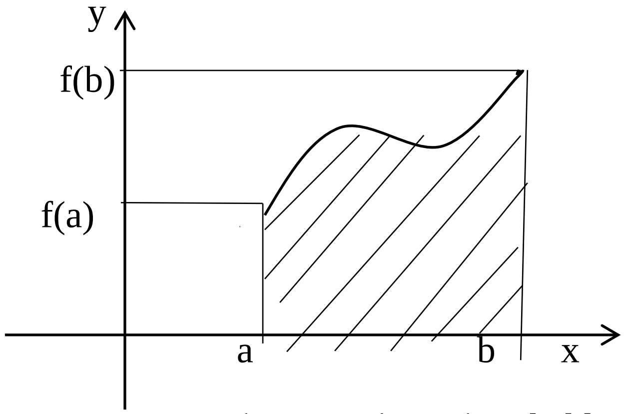
2) $\int x^2 f''(x)dx = \int x^2 d[f'(x)] = \begin{vmatrix} x^2 = u, & d[f'(x)] = dv \\ 2x\,dx = du, & v = f'(x) \end{vmatrix} = x^2 f'(x) - 2\int x\,f'(x)dx = |oldingi\ misolga\ asosan| = x^2 f'(x) - 2[x\,f(x) - F(x)] + C = x^2 f'(x) - 2xf(x) + 2F(x) + C.$

Endigi masala quyidagicha : integral ostidagi $f(x)$ va unga teskari $g(x)$ funksiyalar ma'lum bo'lsa, $\int f(x)dx$ integral quyidagiga teng bo'ladi:

$$\int f(x)dx = x\,f(x) - \int g(x)dx.$$

$\int_a^b f(x)dx$ integral uchun esa:

$\int_a^b f(x)dx = bf(b) - a\,f(a) - \int_{f(a)}^{f(b)} g(x)dx$ formula orqali topiladi. Bu formulaning geometrik tasviri quyidagicha:



b f(b) va $a\,f(a)$ lar to'g'ri to'rtburchaklar yuzalari , $\int_a^b f(x)dx$ va $\int_{f(a)}^{f(b)} g(x)dx$ lar esa egri chiziqli trapetsiyalar yuzalari .

Misol. $\int_1^e lnx\,dx$ integralni hisoblang.

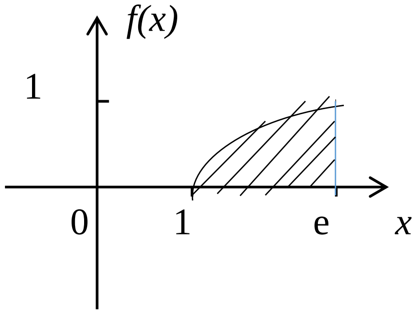
Yechish. Yuqoridagi formulaga asosan:

$$\int_1^e lnx\,dx = e\,lne - 1\,ln1 - \int_0^1 e^x\,dx = 1$$

Agar bu integralni bo'laklab integrallasak,

$$\int_1^e lnx\,dx = \begin{vmatrix} \ln x = u, & dx = dv \\ \frac{dx}{x} = du, & v = x \end{vmatrix} = xln\,x|_1^e - \int_1^e dx = 1$$

Bu misolni geometrik tarzda bayon etaylik. $y = lnx,\ x = 1,\ x = e\ \ va\ y = 0$ chiziqlar bilan chegaralangan figurani yuzini hisoblaylik.



Shtrixlangan figuraning yuzi to'g'ri to'rtburchak yuzidan shtrixlanmagan figurani yuzini ayirmasiga teng bo'ladi.

Teorema. Agar $f(x)$ funksiya $(\alpha;\beta)$ oraliqda differensiallanuvchi va teskarilanuvchi bo'lsa, u holda ixtiyoriy $a \in (\alpha\,;\,\beta)$ uchun

$$F(x) = x\,f(x) - \int_{f(a)}^{f(x)} g(y)d(y)$$

funksiya $f(x)$ funksiya uchun boshlang'ich funksiya bo'ladi.

Agar $u(x)$ va $v(x)$ funksiyalar n marta differensiallanuvchi bo'lsa, u holda

$$\int_a^b u\,v^{(n)}dx = \left[u\ \ v^{(n-1)} - u'v^{(n-2)} + \cdots + (-1)^{n-1}u^{(n-1)}\ v\right]\Big|_a^b + (-1)^n\int_a^b u^{(n)}v\,dx$$

formula o'rinli bo'ladi.

Xususiy holda, agar $n = 0$ bo'lsa, $\int_a^b u\,v\,dx = \int_a^b u\,v\,dx\,,$

$n = 1$ bo'lsa, $\int_a^b u\,v'dx = u\,v - \int_a^b u'v\,dx$

$n = 2$ bo'lsa, $\int_a^b u\,v''dx = (u\,v' - u'v)|_a^b + \int_a^b u''v\,dx$

$n = 3$ bo'lsa, $\int_a^b u\,v'''dx = (u\,v'' - u'v' + u''v)|_a^b - \int_a^b u'''v\,dx$ bo'ladi.

Misol. $\int \frac{x\,dx}{sin^2x}$ integralni hisoblang.

Yechish. $\int \frac{x\,dx}{sin^2 x} = -\int x\,(ctgx)'dx = -[x\,ctg\,x - \int ctg\,x\,dx] = -x\,ctgx + \ln sinx + C.$

Misol. $\int_0^{\pi} x^3 \sin x\,dx$ ni hisoblang.

Yechish. Bu integralni bo'laklab integrallashni n marta qo'llashga to'g'ri keladi va hisoblash ancha vaqtni oladi.

Agar $\int u\,v^{IV}dx = u\,v^{III} - u^{I}\,v^{II} + u^{II}v^{I} - u^{III}v + \int u^{IV}v dx$ formulani tadbiq qilsak,

$\int_0^{\pi} x^3 \sin x\,dx = \int_0^{\pi} x^3(\sin x\,)^{IV}dx == [x^3(-cosx) + 3x^2 sinx + 6x\cos x - 6sinx]|_0^{\pi} = \pi^3 - 6\pi$ ni hosil qilamiz.

Misol. Quyidagi integralni hisoblang:

$$\int_0^1 \ln(x + \sqrt{1 + x^2}\,)dx$$

Yechish. $y = \ln\left(x + \sqrt{1 + x^2}\right)$ funksiyaga teskari funksiyani topamiz.

$x + \sqrt{1 + x^2} = e^y$ deb shakl almashtirib,

$\sqrt{1 + x^2} = e^y - x,\ 1 + x^2 = e^{2y} - 2xe^y + x^2, e^{2y} - 2xe^y - 1 = 0,$

$2xe^y = e^{2y} - 1, x = \frac{1}{2}\left(\frac{e^{2y}-1}{e^y}\right) = \frac{1}{2}(e^y - e^{-y}) = \frac{e^y - e^{-y}}{2}$ ni hosil qilamiz.

Demak , $y = \ln(x + \sqrt{1 + x^2})$ funksiyaga teskari funksiya $y = \frac{e^x - e^{-x}}{2}$ funksiya ekan. Yuqorida keltirilgan formulaga asosan:

$$\int_0^1 \ln\left(x + \sqrt{1 + x^2}\right)dx - \ln\left(1 + \sqrt{2}\right) - \int_0^{\ln\left(1+\sqrt{2}\right)} \frac{e^x - e^{-x}}{2}dx = \ln\left(1 + \sqrt{2}\right) - \frac{1}{2}(\,e^x - e^{-x})|_0^{\ln\left(1+\sqrt{2}\right)}$$
$$= \ln\left(1 + \sqrt{2}\right) - \sqrt{2} + 1$$

kelib chiqadi.

XULOSA

Yuqoridagilardan shunday xulosa kelib chiqadiki, funksiyadan olingan integralni hisoblashda bu funksiyaga teskari bo'lgan sodda funksiyalarga olib kelish masalani yengillashtiradi.

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