VOLUME 2 | ISSUE 2
ISSN 2181-1784
SJIF 2022: 5.947
ASI Factor = 1.7

KO'PHAD VA UNING ILDIZLARI ORASIDAGI MUNOSABAT

Djabbarov Odil Djurayevich
TDTU Olmaliq filiali katta o'qituvchisi
odilxon455@gmail.com
Xujayev Tuymurod Xuddiyevich
TDTU Olmaliq filiali katta o'qituvchisi
tuymurod. khujayev@gmail.com

ANNOTATSIYA

Maqolada ko'phad va uning ildizlari orasidagi munosabatlar o'rganilgan bo'lib, ular uchun umumlashgan Viet teoremasi kiltirilgan. Mavzuga doir misol va masalalar o'rin olgan. Ko'phadni Nyuton binomi formulasi bilan ham aliqadorligi o'rganilgan.

Kalit so'zlar: *Ko'phad, Viet teoremasi, Nyuton binomi, ko'phadning ildizi, koeffisient, tenglama,kombinatorika.*

АННОТАЦИЯ

В статье исследуется связь между многочленом и его корнями, для которых применяется обобщенная теорема Виета. Есть примеры и вопросы по теме. Многочлены также были связаны с биномиальной формулой Ньютона.

Ключевые слова: *многочлен, теорема Виета, бином Ньютона, корень многочлена, коэффициент, уравнение, комбинаторика.*

ABSTRACT

The article examines the relationship between a polynomial and its roots, for which the generalized Viet theorem is applied. There are examples and issues on the topic. Polynomials have also been linked to Newton's binomial formula.

Keywords: *Polynomial, Viet theorem, Newton's binomial, polynomial root, coefficient, equation, combinatorics.*

KIRISH

Bizga ixtiyoriy ratsional koeffisientli n -darajali ko'phad berilgan bo'lsin. Faraz qilaylik u n ta haqiqiy ildizlarga bo'lsin. Bu ildizlarni koeffisientlar bilan qanday bog'langanligini ko'raylik. Quyidagi $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = a_n\,(x - x_1)(x - x_2)\ldots(x - x_n)$ tenglikni o'ng tomonini qavslarini ochib, o'xshash hadlarni mos koeffisentlarini chap tomondagi mos koeffisentlarga tenglashtirib,

$$x_1 + x_2 + x_3 + \cdots + x_n = (-1)\frac{a_{n-1}}{a_n}$$

$$x_1x_2 + x_1x_3 + \cdots + x_{n-1}x_n = (-1)^2\frac{a_{n-2}}{a_n}$$

$$x_1x_2x_3 + x_1x_2x_4 + \cdots + x_{n-2}x_n = (-1)^3\frac{a_{n-3}}{a_n}$$

… …

$$x_1x_2 \dots x_n = (-1)^n\frac{a_0}{a_n}$$

munosabatni hosil qilamiz. Bu formula umumlashgan Viet formulasi deyiladi.

Masalan, $ax^2 + bx + c = 0$ kvadrat tenglama uchun:

$$\begin{cases} x_1 + x_2 = -\frac{b}{a} \\ x_1\ x_2 = \frac{a}{c} \end{cases}$$

MUHOKAMA VA NATIJALAR

Teorema. Agar *a* son $P_n(x) = 0$ tenglamaning ildizi bo'lsa, u holda $\frac{P_n(1)}{a-1}$ va $\frac{P_n(-1)}{a+1}$ butun sonlar bo'ladi.

Agar 1 va -1 sonlar ko'phadning ildizlari bo'lsa, uni *x-1* va *x+1* ga bo'lib, bo'linma ko'phad uchun teoremani qo'llaymiz.

Misol. $2x^4 - 13x^2 + 13x - 6 = 0$ tenglamani butun ildizlarini ozod hadning bo'luvchilarini $\pm1, \pm2, \pm3, \pm6$ sonlar ichidan izlaymiz.

$P_4(1) = 2 - 13 + 13 - 6 = -4$, $P_4(-1) = 2 - 13 - 13 - 6 = -30$, $P_4(2) = 32 - 52 + 26 - 6 = 0$

.

Misol. x^n + $x^{n-1} + x^{n-2}$ +...+x+n = 0 tenglama rasional ildizlarga ega emasligini isbotlang, bu yerda n – tub son.

Isbot. Tenglamaning ildizi n- ning bo'luvchilari ±1 va $\pm n$ ekanligini bilib,

f(1) = n+n $\neq 0$, f(-1) = n-1 $\neq 0$, f(n) = n^n+$n^{n-1} + n^{n-2}$+...+ n + n $\neq 0$ va f(-n) = $-n^2(n^{n-2}$ - $n^{n-3} + n^{n-2}$ +...+n-1) $\neq 0$. Demak, bundan ko'rinadiki, tenglama rasional ildizlarga ega emas ekan.

Teorema. Agar juft darajali ko'phadning juft o'rindagi hadlar koeffisentlari toq o'rindagi hadlar koeffisentlariga teng bo'lsa, u holda $x = -1$ ko'phad ildizi bo'ladi.

Masalan, $P_3(x) = 3x^3 + 7x^2 - 4x - 8$ ko'phad uchun 3-4=7-8.

U holda $P_3(-1) = -3 + 7 + 4 - 8 = 0$ bo'ladi.

VOLUME 2 | ISSUE 2
ISSN 2181-1784
SJIF 2022: 5.947
ASI Factor = 1.7

$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ $a_n \neq 0$ ko'phad uchun

$P(1) = a_n + a_{n-1} + \cdots + a_1 + a_0$

$P(-1) = (-1)^n a_n + (-1)^{n-1} a_{n-1} + \cdots + a_1 + a_0$ bo'ladi va

$\frac{P(1)+P(-1)}{2}$ son ko'phadning juft o'rindagi hadlarning koeffisentlari yig'indisini beradi.

Misol. $P_3(x) = 2x^3 - 4x^2 + 5x + 1$ ko'phad uchun $P(-1) = 2 - 4 + 5 + 1 = 4$, $P(-1) = -2 - 4 - 5 + 1 = -10$.

Ulardan $\frac{P(1)+P(-1)}{2} = \frac{4-10}{2} = -3$, $\frac{P(1)+P(-1)}{2} = \frac{4+10}{2} = 7$.

Haqiqatdan ham, $2 + 5 = 7, -4 + 1 = -3$.

$P(1)$ esa ko'phadning barcha hadlari koeffisentlari yig'indisini beradi.

$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ ko'phad algebraning asosiy teoremasiga ko'ra ko'pi bilan n ta haqiqiy ildizga ega. Faraz qilaylik, ko'phad n ta $x_1, x_2, \ldots, x_n$ ta ildizlarga ega bo'lsin. U holda ko'phadning ko'rinishi quyidagicha bo'ladi:

$P_n(x) = a_n(x - x_1)(x - x_2)\ldots(x - x_n)$

Quyidagi masalalarni ko'raylik:

Agar $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ ko'phadning ildizlari $x_1, x_2, \ldots, x_n$ bo'lsa, quyidagi ko'phadlarning ildizlari qanday bo'ladi:

1. $a_n x^n - a_{n-1}x^{n-1} + \cdots + (-1)^{n-1} a_n x + (-1)^n_n a_0$
2. $a_0 x^n + a_1 x^{n-1} + \cdots + a_n$
3. $a_n x^n - a_{n-1} b x^{n-1} + \cdots + a_0 b^n$

Bu ko'phadlarning ildizlari 1) $-x_1, -x_2, \ldots, -x_n$ 2) $\frac{1}{x_1}, \frac{1}{x_2}, \ldots, \frac{1}{x_n}$

3) $bx_1, bx_2, \ldots, bx_n$ bo'ladi.

Buni misollarda ko'ratylik.

Misol. $P_2(x) = 2x^2 - 5x + 4$ ko'phadning ildizlari $x_1 = 1,\ x_2 = 4$ ni bilgan holda

1) $P_2(x) = 2x^2 - 5x + 4$ ko'phadni ildizlari $x_1 = -1,\ x_2 = -4$ bo'ladi.

2) $P_2(x) = 2x^2 - 5x + 1$ ko'phadni ildizlari $x_1 = -1,\ x_2 = -\frac{1}{4}$ bo'ladi.

3) $b = 3$ bo'lgan holda $P_2(x) = x^2 - 15x + 36$ ko'phadning ildizlari $x_1 = 3,\ x_2 = 12$ bo'ladi.

Misol. $x^n + a_1x^{n-1} + a_2x^{n-2}$ +...+ a_{n-1} x+ a_n ko'phadning ildizlari kvadratlarining yig'indisini toping.

Yechish.Viet teoremasiga asosan:

$$x_1 + x_2 + x_3 + \cdots + x_n = a_1,$$

$$x_1x_2 + x_1x_3 + \cdots + x_{n-1}x_n = -a_2.$$

Birinchi munosabatni kvadratga ko'tarib: $(x_1 + x_2 + x_3 + \cdots + x_n)^2 = {x_1}^2+{x_2}^2+{x_3}^2+...+{x_n}^2 +$ $2(x_1x_2 + x_1x_3 + \cdots + x_{n-1}x_n)$ ni hosil qilamiz. Bundan ${x_1}^2 + {x_2}^2 + {x_3}^2 + \ldots + {x_n}^2 = {a_1}^2 - 2a_2$ ni topamiz.

Agar ko'phadning barcha ildizlari bir-biriga teng bo'lsa, u quyidagi ko'rinishga ega bo'ladi: $(x + a)^n$.

Quyidagi x+a ikkihadni n$\in N \cup \{0\}$darajalarini ko'rib chiqaylik. Ular uchun quyidagicha jadval o'rinli:

$(x + a)^o$ 1 $\binom{0}{0}$

$(x + a)^1$ 1 1 $\binom{0}{1}$ $\binom{1}{1}$

$(x + a)^2$ 1 2 1 $\binom{0}{2}$ $\binom{1}{2}$ $\binom{2}{2}$

... ...

$(x + a)^n$ 1 n ... ... n 1 $\binom{0}{n}$ $\binom{1}{n}$ ……….. $\binom{n-1}{n}$ $\binom{n}{n}$

$(x + a)^n$ ikkihadni koeffisientlaridan tuzilgan jadval Paskal uchburchagi deyiladi. Bu ikkihadning n-chi darajasi Nyuton binomi deyiladi va u ushbu ko'rinishda bo'ladi: $(x + a)^n = \binom{0}{n}x^n a^o + \binom{1}{n}x^{n-1}a^1 + ... + \binom{n-1}{n}x^1a^{n-1} + \binom{n}{n}x^0a^n$, bu yerda $\binom{0}{n}$, $\binom{1}{n}$,... $\binom{n}{n}$ – binom koeffisientlari deb ataladi. Bu formuladan keltirib chiqariladigan ayrim kombinatorik masalalarni ko'rib chiqaylik. Agar formulada

1) *x=1, a=1* bo'lsa, $2^n = \binom{0}{n} + \binom{1}{n} + ...\binom{n-1}{n} + \binom{n}{n}$ munosabat kelib chiqadi.

2) *x =1, a = -1* bo'lsa, $\binom{0}{n} - \binom{1}{n} + \binom{2}{n} - \binom{3}{n} + \cdots (-1)^k\binom{k}{n} + \cdots + (-1)^n\binom{n}{n}$ = 0 bo'ladi. Bundan esa $\binom{0}{n} + \binom{2}{n} + \cdots = \binom{1}{n} + \binom{3}{n} + \cdots$ kelib chiqadi. *x* va *a* ga ixtiyoriy butun sonlarni qo'yib, boshqa kombinatorik munosabatlarni keltirib chiqarish mumkin.Shulardan ayrimlari bilan tanishib chiqaylik.

1-masala. Quyidagi munosabatni o'rinli ekanligini isbotlang:

$$1- 3^1\binom{1}{4k} + 3^2\binom{2}{4k} - 3^3\binom{3}{4k} + ... + 3^{4k-2}\binom{2}{4k} - 3^{4k-1}\binom{1}{4k} + 3^{4k}\binom{0}{4k}$$

$$=1-5^1\binom{1}{2k} + 5^2\binom{2}{2k} - 5^3\binom{3}{2k} + ... + 5^{2k-2}\binom{2}{2k} - 5^{2k-1}\binom{1}{2k} + 5^{2k}\binom{0}{2k}.$$

Isbot. Nyuton binomi formulasiga asosan: $(1-3)^{4k}\ va\ (1-5)^{2k}$ ikkihadlar uchun: $(1-3)^{4k} = (1-5)^{2k}$ munosabatdan masala to'g'riligi kelib chiqadi.

2- masala. Isbotlang: $\binom{0}{n} + 2^1\binom{1}{n}+2^2\binom{2}{n}+\ldots+2^n\binom{n}{n}=3^n$.

Isbot. $(1+2x)^n = \binom{0}{n}(2x)^0 + \binom{1}{n}(2x)^1+\binom{2}{n}(2x)^2 +\ldots+ \binom{n}{n}(2x)^n$ formuladan foydalanib, *x= 1* da hisoblaymiz: $3^n = \binom{0}{n} + 2^1\binom{1}{n}+2^2\binom{2}{n}+\ldots+2^n\binom{n}{n}$.

3-masala. $\binom{0}{n}^2 + \binom{1}{n}^2 + \binom{2}{n}^2 +\ldots+\binom{n}{n}^2=\binom{n}{2n}$ ekanligini isbotlang.

Isbot. $(1+x)^n(1+x)^n = (1+x)^{2n}$ formulaga asosan:

$$(\binom{0}{n}x^0+\binom{1}{n}x^1+\binom{2}{n}x^2+\ldots\binom{n}{n}x^n)(\binom{0}{n}x^0+\binom{1}{n}x^1+\binom{2}{n}x^2+\ldots\binom{n}{n}x^n)=$$
$$=\binom{0}{2n}x^0+\binom{1}{2n}x^1+\binom{2}{2n}x^2+\ldots+\binom{2n}{2n}x^{2n}.$$

x^n ning koeffisienti quyidagiga teng:

$$\binom{0}{n}\binom{n}{n}+\binom{1}{n}\binom{n-1}{n}+\binom{2}{n}\binom{n-2}{n}+\ldots+\binom{n}{n}\binom{0}{n}.$$

Ma'lumki, $\binom{n-k}{n} = \binom{k}{n}$ tenglik o'rinli bo'lganligi sababli , x^n ning koeffisienti

$$\binom{0}{n}^2 + \binom{1}{n}^2 + \binom{2}{n}^2 +\ldots+\binom{n}{n}^2.$$

U xolda $\binom{0}{n}^2 + \binom{1}{n}^2 + \binom{2}{n}^2 +\ldots+\binom{n}{n}^2=\binom{n}{2n}$ kelib chiqadi.

4-masala. $\frac{1}{1}\binom{0}{n} + \frac{1}{2}\binom{1}{n} + \frac{1}{3}\binom{2}{n} +\ldots+\frac{1}{n+1}\binom{n}{n}$ yig'indini toping.

Yechish. $\frac{\binom{k}{n}}{k+1} = \frac{\binom{k+1}{n+1}}{n+1}$ formuladan foydalanib, k=0,1,2,...,n deb olib, $\frac{\binom{0}{n}}{1} = \frac{\binom{1}{n+1}}{n+1}, \frac{\binom{1}{n}}{2} = \frac{\binom{2}{n+1}}{n+1}, \ldots, \frac{\binom{n-1}{n}}{n} = \frac{\binom{n}{n+1}}{n+1}, \frac{\binom{n}{n}}{n+1} = \frac{\binom{n+1}{n+1}}{n+1}$ larni hadma-had qo'shib

$$\frac{1}{1}\binom{0}{n} + \frac{1}{2}\binom{1}{n} + \frac{1}{3}\binom{2}{n} +\ldots+\frac{1}{n+1}\binom{n}{n} =$$
$$= \frac{1}{n+1}\left[\binom{1}{n+1} + \binom{2}{n+1} + \cdots + \binom{n+1}{n+1}\right] =$$
$$\frac{1}{n+1}\left[-\binom{0}{n+1} + \binom{0}{n+1} + \binom{1}{n+1} + \binom{2}{n+1} + + \cdots + \binom{n+1}{n+1}\right] = \frac{1}{n+1}[2^{n+1}-1]$$

ni hosil qilamiz.

5-masala. S_1 =1- $\binom{2}{n} + \binom{4}{n} - \binom{6}{n} + \cdots\ \ va\ \ S_2 = \binom{1}{n} - \binom{3}{n} + \binom{5}{n} - \binom{7}{n} + \cdots$ yig'indini toping.

Yechish. Bu masalani yechish uchun kompleks sonni darajaga ko'tarish formulasidan foydalanamiz. Xususiy xol uchun

$$(1+i)^n = S_1+ iS_2=\left[\sqrt{2}(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right]^n = 2^{\frac{n}{2}}\left(cos\frac{n\pi}{4} + isin\frac{n\pi}{4}\right) =$$
$$= 2^{\frac{n}{2}}cos\frac{n\pi}{4} + i2^{\frac{n}{2}}sin\frac{n\pi}{4}$$

ni hosil qilamiz. Bu yerdan

$$S_1 = 2^{\frac{n}{2}} cos\frac{n\pi}{4} \text{ va } S_2 = 2^{\frac{n}{2}} sin\frac{n\pi}{4}$$

kelib chiqadi.

6-masala. $sinx + sin2x + \cdots + sinnx = \frac{sin\frac{n+1}{2}x\, sin\frac{nx}{2}}{sin\frac{x}{2}}$ ni isbotlang.

Isbot. $T = \sum_{k=1}^{n} sinkx$ va $S = \sum_{k=1}^{n} coskx$ deb olib, $S + iT = \sum_{k=1}^{n}(coskx + isinkx) = \sum_{k=1}^{n}(cos\frac{x}{2} + isin\frac{x}{2})^{2k} = \sum_{k=1}^{n} \alpha^{2k} = \frac{\alpha^n(\alpha^{n+1} - \frac{1}{\alpha^{n+1}})}{\alpha - \frac{1}{\alpha}}$

ni hosil qilamiz. Bundan $T = \frac{sin\frac{n+1}{2}xsin\frac{nx}{2}}{sin\frac{x}{2}}$ kelib chiqadi.

XULOSA

Yuqoridagilardan shunday xulosa kelib chiqadiki, ko'phad va uning ildizlari orasidagi munosabat, ya'ni Viet teoremasi juda ko'plab masalalarni yechishda qo'llash mumkin ekan. Bu munosabat Nyuton formulasi bilan ham bog'liq ekanligini ko'rish mumkin.

REFERENCES

1. Djabbarov. O.Dj., & Iskandarov. S. D. (2021). TEYLOR FORMULASI VA UNING TURLI MATEMATIK MASALALARGA QO'LLANILISHI. *Oriental renaissance: Innovative, educational, natural and social sciences*. 1(3), 773-778.

2. Djabbarov.O.Dj. & Akbaraliyev. A. A. (2021) O'RTA ARIFMETIK VA O'RTA GEOMETRIK TUSHUNCHAGA BOG'LIQ KETMA-KETLIKLAR LIMITI , *Oriental renaissance: Innovative, educational, natural and social sciences*. 1(1). 94-97.

3. Djabbarov. O. Dj. & Jabborxonova. G. (2021) DARAXT HAJMINI HISOBLASHNING BIR MATEMATIK USULI. *Oriental renaissance: Innovative, educational, natural and social sciences*. 1(1). 249-252.

4. Djabbarov. O.Dj. & Jabborxonova. G. (2021) MATEMATIK O'ZGARMASLARNING TURLI KO'RINISHLARI HAQIDA. *Oriental renaissance: Innovative, educational, natural and social sciences*. 1(2). 237-240

5. Djabbarov. O.Dj. & Abdiashimova. M. (2021) MATEMATIKA FANINI O'RGANISHDA QIZIQARLI MASALALARNING O'RNI HAQIDA. *Oriental renaissance: Innovative, educational, natural and social sciences*, 1(2). 233-236.